

# ECE2 covariant universal gravitation and precession

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## 3 Numerical and graphical development

The orbital precession in near-circular approximation was given by Eq. (60). By the relations (57-59) for circular orbits, only the constants  $\alpha$  (half right latitude) and  $a$  (semi major axis) are left as input parameters. The integral depends on the quadratic mean fluctuation radius  $\langle \delta \mathbf{r} \cdot \delta \mathbf{r} \rangle$ . If we assume that this is constant, we can compute the precession angle per quadratic fluctuation:

$$\frac{\Delta \phi}{\langle \delta \mathbf{r} \cdot \delta \mathbf{r} \rangle} \approx \frac{2}{3} \sqrt{\alpha} \int_{u_{min}}^{u_{max}} \frac{u^3 \log(u)}{(-\alpha u^2 + 2u + \frac{1}{a})^{\frac{3}{2}}} du \quad (63)$$

where  $u = 1/r$  is the inverse radius. The minimum and maximum radius are

$$r_{min} = a(1 - \epsilon), \quad (64)$$

$$r_{max} = a(1 + \epsilon). \quad (65)$$

The semi major axis is

$$a = \frac{\alpha}{1 - \epsilon^2} \quad (66)$$

from which the bounds of integration follow:

$$u_{min} = \frac{1}{r_{max}} = \frac{1 - \epsilon^2}{\alpha(\epsilon + 1)} = \frac{1 - \epsilon}{\alpha}, \quad (67)$$

$$u_{max} = \frac{1}{r_{min}} = \frac{1 - \epsilon^2}{\alpha(1 - \epsilon)} = \frac{1 + \epsilon}{\alpha}. \quad (68)$$

We carried out numerical solutions of the integral (63), using a model system with  $\alpha = 1$  and  $\epsilon = 0.3$ . The integrand has been graphed in dependence of  $u$  in Fig. 1. As can be seen, it is regular over the full range between  $u_{min}$  and  $u_{max}$  although the function has poles at other values of  $u$ . So a numerical integration

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is straightforward. The ratio  $\Delta\phi/\langle\delta\mathbf{r}\cdot\delta\mathbf{r}\rangle$  is shown in Fig. 2 in dependence of the eccentricity  $\epsilon$ . For clarity the lower range has been magnified by a factor of 10. The curve raises similar to an exponential function. It should be noted that the range exceeds the region of a near-circular orbit but is selected here to demonstrate the behaviour more clearly.

Finally we calculate the isotropically averaged vacuum fluctuation radius for the planet Mercury. The precession angle per orbit is (see UFT 391):

$$\Delta\phi = 5.019 \cdot 10^{-7} \text{ rad.} \quad (69)$$

From Eq. (63) follows with  $a = 57,909,050$  km and  $\epsilon = 0.205630$ :

$$\frac{\Delta\phi}{\langle\delta\mathbf{r}\cdot\delta\mathbf{r}\rangle} = -8.47415 \cdot 10^{-22} \frac{\text{rad}}{\text{m}^2}. \quad (70)$$

Taking the modulus of this value, this gives a fluctuation radius of

$$\langle\delta r\rangle = \sqrt{\langle\delta\mathbf{r}\cdot\delta\mathbf{r}\rangle} = 24.3366 \text{ km} \quad (71)$$

which is two orders of magnitude smaller than the Schwartzschild radius of the sun.

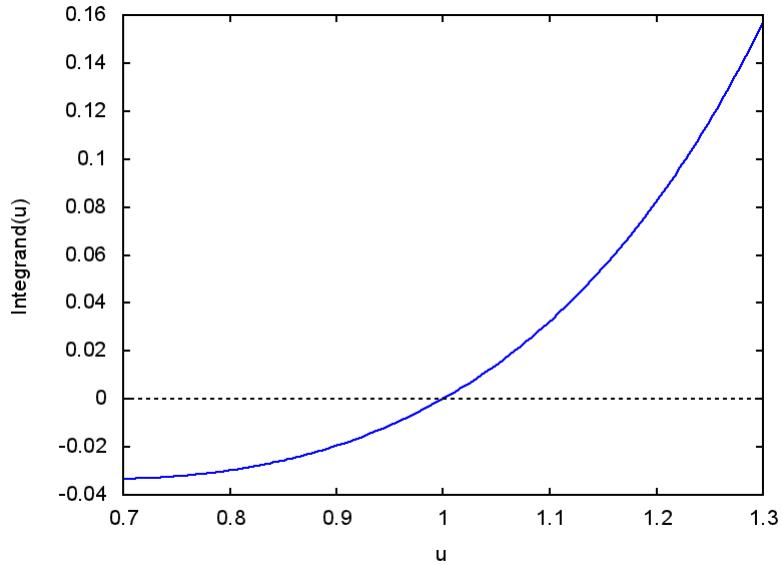


Figure 1: Integrand of Eq. (63) for a model system.

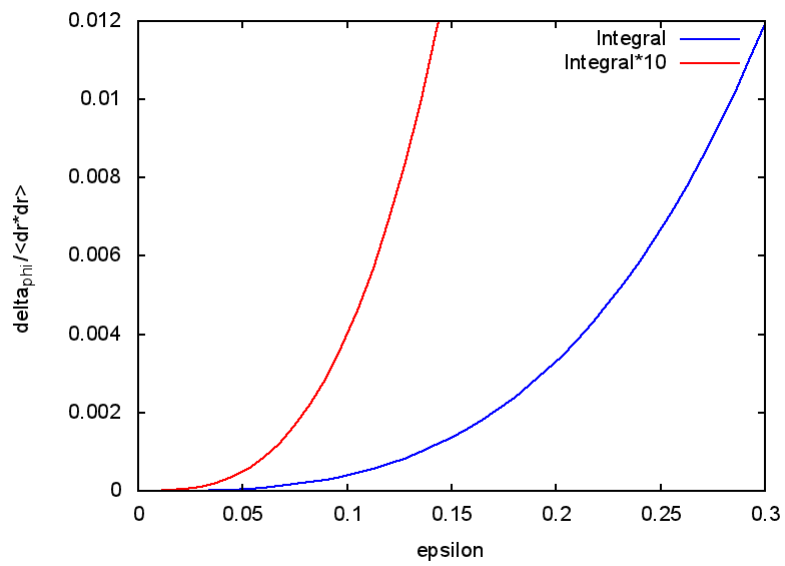


Figure 2: Ratio  $\Delta\phi / \langle \delta\mathbf{r} \cdot \delta\mathbf{r} \rangle$  in dependence of orbital parameter  $\epsilon$ .