

```
(%i1) kill(all);
(%o0) done
```

```
(%i1) assume (u>0, alpha>0, a>0);
(%o1) [u>0, alpha>0, a>0]
```

1 Integration for Delta phi

```
(%i2) I_delta_phi: 2/3*drdr*log(u)*(u^2/(-alpha*u^2+2*u+1/a))^(3/2);
(%o2) 
$$\frac{2 \operatorname{drdr} u^3 \log(u)}{3 \left(-\alpha u^2 + 2 u + \frac{1}{a}\right)^{3/2}}$$

```

```
(%i3) integrate(I_delta_phi,u);
(%o3) 
$$\frac{2 \operatorname{drdr} \int \frac{u^3 \log(u)}{\left(-\alpha u^2 + 2 u + \frac{1}{a}\right)^{3/2}} du}{3}$$

```

1.1 Simplified test integrands

```
(%i4) I2_delta_phi: 2/3*drdr*(1/(-alpha*u^2+2*u+1/a))^(3/2);
(%o4) 
$$\frac{2 \operatorname{drdr}}{3 \left(-\alpha u^2 + 2 u + \frac{1}{a}\right)^{3/2}}$$

```

```
(%i5) integrate(I2_delta_phi,u);
(%o5) 
$$\frac{2 \operatorname{drdr} \left( \frac{4}{\left(-\frac{4\alpha}{a} - 4\right) \sqrt{-\alpha u^2 + 2 u + \frac{1}{a}}} - \frac{4\alpha u}{\left(-\frac{4\alpha}{a} - 4\right) \sqrt{-\alpha u^2 + 2 u + \frac{1}{a}}} \right)}{3}$$

```

```
(%i6) I2_delta_phi: (alpha/(-alpha*u^2+2*u+1/a))^(1/2);
(%o6) 
$$\frac{\alpha}{\sqrt{-\alpha u^2 + 2 u + \frac{1}{a}}}$$

```

```
(%i7) integrate(I2_delta_phi,u);
(%o7) 
$$-\operatorname{asin}\left(\frac{2 - 2\alpha u}{\sqrt{\frac{4\alpha}{a} + 4}}\right)$$

```

2 Numerical integration

2.1 Integration bounds

```
(%i8) a: alpha/(1-epsilon^2);
```

$$\frac{\alpha}{1-\epsilon^2}$$

```
(%i10) umin1: 1/(a*(1+epsilon));
umax1: 1/(a*(1-epsilon));
```

$$\frac{1-\epsilon^2}{\alpha(\epsilon+1)}$$

$$\frac{1-\epsilon^2}{\alpha(1-\epsilon)}$$

```
(%i12) umin: (1-epsilon)/alpha;
umax: (1+epsilon)/alpha;
```

$$\frac{1-\epsilon}{\alpha}$$

$$\frac{\epsilon+1}{\alpha}$$

```
(%i14) ratsimp(umin-umin1);
ratsimp(umax-umax1);
```

$$0$$

$$0$$

□ 2.2 Integration

```
(%i15) str: [alpha=1, drdr=1, epsilon=0.3];
```

$$[\alpha=1, drdr=1, \epsilon=0.3]$$

```
(%i16) I1: ev(I_delta_phi, str, eval);
```

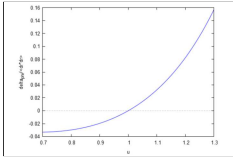
$$\frac{2 u^3 \log(u)}{3 (-u^2 + 2 u + 0.91)^{3/2}}$$

```
(%i18) u1: ev(umin, str);
u2: ev(umax, str);
```

$$0.7$$

$$1.3$$

```
(%i19) wxplot2d([I1], [u,u1, u2], [ylabel, "delta_{phi}/<dr*dr>"])$
```



```
(%t19)
```

```
(%i20) plot2d([I1], [u,u1, u2], [ylabel, "delta_{phi}/<dr*dr>"],
              [gnuplot_term, "png linewidth 2 font 'Arial' 16 size 800,600"],
              [gnuplot_out_file, "D:/Doc/Artikel-Eck/ECE-Theorie/Paper403/Fig1"])
```

```
(%i21) quad_qags(I1, u, u1, u2);
```

```
(%o21) [0.01193310391636642, 2.825072724151846 10-16, 21, 0]
```

□ **3 Eq. (27)**

```
(%i22) delta_phi: 4/3*((1-epsilon)/alpha)^2*drdr;
```

```
(%o22) 
$$\frac{4 \text{ drdr } (1 - \epsilon)^2}{3 \alpha^2}$$

```

```
(%i23) ev(delta_phi, str);
```

```
(%o23) 0.6533333333333332
```