ANALYTICAL ORBITAL EQUATION FOR ECE2 COVARIANT PRECESSION.

by

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ABSTRACT

An analytical expression is obtained for ECE2 covariant forward and retrograde precession. This is the ECE2 covariant Binet equation, valid for all force laws. For an inverse square law it gives the orbit from the relativistic Newton equation. The handedness of the precession depends on the sign of the spin connection, which originates in the mean square fluctuation of the vacuum

Keywords: ECE2 relativistic Binet equation, forward and retrograde precession.

UFT 402

1. INTRODUCTION

Recently in this series of over four hundred papers and books {1 - 41} it has been shown that the origin of orbital precession is the mean square fluctuation of the vacuum, so orbital and Lamb shift theory can be understood within the framework of the ECE and ECE2 unified field theories. In the immediately preceding paper UFT401 analytical integration of the ECE2 covariant Newton equation was used to show that it gives both forward and retrograde precession, depending on the sign of the spin connection. The latter has been shown to originate in vacuum fluctuations of the type used in the well known Lamb shift theory.

In section 2, the relativistic Newton equation is transformed into the relativistic Leibniz equation and the equation of conservation of relativistic angular momentum. The former is further transformed into the relativistic Binet equation, which when integrated gives the relativistic orbit for any force law. This is an analytical procedure which complements the numerical procedures of UFT401 and preceding papers. Section 3 is a discussion of results, with graphics.

This paper is a brief synopsis of extensive calculations contained in the notes accompanying UFT402 on www.aias.us. These notes are an intrinsic part of the paper and should be read with the paper. Note 402(1) discusses the origin of forward and retrograde precession in the ECE2 covariant spin connection and isotropic vacuum fluctuations of the type used in Lamb shift theory. Notes 402(2) and 402(3) discuss the derivation of the force equation from the lagrangian, defining the generalized momentum. Note 402(4) is a brief review of the origin of orbital precession in vacuum fluctuations. Note 402(5) is the proof of conservation of relativistic angular momentum. Note 402(6) is the detailed definition of the relativistic lagrangian and an overview of Newtonian dynamics, and Notes 402(7) gives all detail of the derivation of the relativistic Binet equation of orbits, valid for any force law.

2. THE ECE2 COVARIANT BINET EQUATION

Consider the ECE2 covariant Newton equation {1 - 41} of orbits:

$$\underline{F} = \gamma^3 m \ddot{\underline{r}} = -mMG\frac{\underline{r}}{r^3} \quad -(1)$$

in which an object of mass m orbits an object of mass M, attracted by the inverse square law in Eq. (1). Here $\underline{F}$ is the force, G is Newton's constant, $\underline{r}$ is the position vector joining m and M, and

$$\gamma^3 = \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad -(2)$$

where $\underline{v}$ is the orbital velocity and c the speed of light. In ECE2 theory the force is defined by:

$$\underline{F} = -\underline{\nabla}\phi + \underline{\omega}\phi \quad -(3)$$

where

$$\phi = -\frac{mMG}{r} \quad -(4)$$

is the gravitational potential and $\underline{\omega}$ is the vector spin connection. It follows that:

$$\underline{\omega} = \frac{\underline{r}}{r^2}\left(\frac{1}{\gamma^3} - 1\right) \quad -(5)$$

with magnitude:

$$\omega = |\underline{\omega}|. \quad -(6)$$

In UFT401 it was shown that the handedness of the precession of the orbit is defined by the sign of the spin connection, a major advance over the standard model.

The force due to the vacuum is:

$$\underline{F}(\mathrm{vac}) = \underline{\omega}\,\phi = -\frac{mMG}{r}\,\underline{\omega} \quad -(7)$$

and as shown in UFT401 the magnitude of the spin connection is defined by:

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle = \frac{3}{2} r^3 \omega = \frac{3}{2} r^2 \left(1 - \left(1 - \frac{v^2}{c^2} \right)^{3/2} \right) \quad -(8)$$

where $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ is the isotropically averaged square of the vacuum fluctuation $\delta \underline{r}$ in the position vector. The well known Lamb shift theory uses the same concept $\delta \underline{r}$. In the first approximation (for small precessions) the Newtonian orbital velocity:

$$v^2 = mG \left(\frac{2}{r} - \frac{1}{a} \right) \quad -(9)$$

can be used. Here a is the semi major axis:

$$a = \frac{\alpha}{1 - \epsilon^2} \quad -(10)$$

where α and ϵ are the astronomically measured and tabulated half right latitude and eccentricity of any orbit. Therefore the mean square fluctuation can be deduced for any orbit. Conversely, a given mean square fluctuation results in a particular orbit.

Similarly the angular frequency of the vacuum fluctuation is given in UFT401 as:

$$\Omega_0^2 = \frac{2}{3} \frac{mG}{r^4} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle^{1/2} = \left(\frac{2}{3} \right)^{1/2} \frac{mG}{r^3} \left(1 - \left(1 - \frac{v^2}{c^2} \right)^{3/2} \right)^{1/2} \quad -(11)$$

and can be calculated for any given α and ϵ.

Consider the well known relativistic velocity:

$$\underline{v} = \gamma \frac{d\underline{r}}{dt} \quad -(12)$$

It follows that the relativistic acceleration is:

$$\underline{a} = \frac{d}{dt}(\gamma \dot{\underline{r}}) = \gamma \ddot{\underline{r}} + \dot{r}\frac{d\gamma}{dt} \quad -(13)$$

and that the relativistic force equation (1) can be written as:

$$\underline{F} = m\left(\gamma \ddot{\underline{r}} + \dot{\underline{r}}\frac{d\gamma}{dt}\right) = -\frac{mMG}{r^3}\underline{r} \quad -(14)$$

In plane polar coordinates r and :

$$\ddot{\underline{r}} = (\ddot{r} - r\dot{\phi}^2)\underline{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\underline{e}_\phi \quad -(15)$$

and

$$\dot{\underline{r}} = \dot{r}\underline{e}_r + r\dot{\phi}\underline{e}_\phi \quad -(16)$$

so Eq. (14) gives the relativistic Leibniz equation:

$$\gamma(\ddot{r} + r\dot{\phi}^2) + \frac{d\gamma}{dt}\dot{r} = -\frac{MG}{r^2} \quad (17)$$

and the equation:

$$\gamma(r\ddot{\phi} + 2\dot{r}\dot{\phi}) + r\dot{\phi}\frac{d\gamma}{dt} = 0 \quad -(18)$$

From a lagrangian analysis given in previous UFT papers, the relativistic angular momentum is defined as:

$$L = \gamma m r^2 \dot{\phi} \quad -(19)$$

and is a constant of motion in an ECE2 covariant theory, i.e. is a conserved quantity:

$$\frac{dL}{dt} = 0. \quad -(20)$$

It follows that:

$$\frac{d}{dt}\left(\gamma m r^2 \dot{\phi}\right)$$

$$= \gamma\left(r^2 \ddot{\phi} + \frac{dr^2}{dt}\dot{\phi}\right) + r^2 \dot{\phi}\frac{d\gamma}{dt} = 0 \quad -(21)$$

Now use:

$$\frac{dr^2}{dt} = \frac{dr^2}{dr}\frac{dr}{dt} = 2\dot{r}r \quad -(22)$$

and Eq. (21) becomes:

$$\gamma\left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right) + r\dot{\phi}\frac{d\gamma}{dt} = 0 \quad -(23)$$

which is Eq. (18), Q. E. D.

Therefore the ECE2 covariant force equation (14) rigorously conserves the relativistic angular momentum. The hamiltonian of the theory is:

$$H = \gamma m c^2 - \frac{mMG}{r^2} \quad -(24)$$

and is also a conserved constant of motion:

$$\frac{dH}{dt} = 0. \quad -(25)$$

The lagrangian of the theory is:

$$\mathcal{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r^2} \quad -(26)$$

in which the velocity is defined as:

$$v^2 = v_x^2 + v_y^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(27)$$

The Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) \quad -(28)$$

gives the relativistic Leibniz equation (17), and the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) \quad -(29)$$

gives:

$$\frac{dL}{dt} = 0, \quad L = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \gamma m r^2 \dot{\phi}. \quad -(30)$$

The generalized momentum:

$$p = \frac{\partial \mathcal{L}}{\partial \dot{r}} = \gamma m \dot{r} \quad -(31)$$

is the relativistic linear momentum. The generalized momentum:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \gamma m r^2 \dot{\phi} \quad -(32)$$

is the relativistic angular momentum.

For the analytical calculation of orbits it is an advantage to transform the Leibniz equation into the well known Binet equation {1 - 41}:

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2} F(r) \quad -(33)$$

the details of this transformation are given in Note 402(7) and in many textbooks. On the non relativistic level the transformation uses the change of variable:

$$u = \frac{1}{r} \quad -(34)$$

and the angular momentum:

$$L = mr^2 \frac{d\phi}{dt} \quad -(35)$$

The Binet equation (33) is valid for any force law:

$$F(r) = -\frac{\partial U(r)}{\partial r} \quad -(36)$$

For the inverse square law of attraction between m and M:

$$F(r) = -\frac{mMG}{r^2} \quad -(37)$$

the non relativistic Binet equation is:

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} \quad -(38)$$

where:

$$\alpha = \frac{L^2}{m^2 MG} \quad -(39)$$

is the half right latitude. It is seen by inspection that the conic section:

$$r = \frac{\alpha}{1 + \epsilon \cos\phi} \quad -(40)$$

is a solution of the Binet equation, Q. E. D.

Now consider the relativistic Leibniz equation (17):

$$\gamma\left(\ddot{r} - r\dot{\phi}^2\right) + \dot{r}\frac{d\gamma}{dt} = -\frac{MG}{r^2} \quad -(41)$$

and note that:

$$\frac{d}{dt}(\gamma\dot{r}) = \gamma\ddot{r} + \dot{r}\frac{d\gamma}{dt} \quad -(42)$$

From Eq. (19):

$$\frac{d\phi}{dt} = \frac{L}{\gamma m r^2} \quad -(43)$$

so:

$$\frac{du}{d\phi} = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\phi} = -\frac{\gamma m}{L}\dot{r}. \quad -(44)$$

It follows that:

$$\frac{d^2 u}{d\phi^2} = -\frac{m}{L}\frac{d}{d\phi}\left(\gamma\dot{r}\right) = -\frac{m}{L}\frac{dt}{d\phi}\frac{d}{dt}\left(\gamma\dot{r}\right) = -\frac{\gamma m^2 r^2}{L^2}\frac{d}{dt}\left(\gamma\dot{r}\right). \quad -(45)$$

Therefore:

$$\frac{d}{dt}\left(\gamma\dot{r}\right) = -\frac{L^2}{\gamma m^2 r^2}\frac{d^2 u}{d\phi^2} \quad -(46)$$

and:

$$\gamma r\dot{\phi}^2 = -\frac{L^2}{\gamma m^2 r^3} \quad -(47)$$

and the relativistic Leibniz equation (41) becomes the relativistic Binet equation of orbits

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\gamma m r^2}{L^2}F(r) \quad -(48)$$

This is the ECE2 covariant equation of orbits, Q. E. D. It is valid for any force law, and for an inverse square force law becomes:

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{\gamma}{\alpha}. \quad -(49)$$

These results confirm other derivations given in previous UFT papers, showing complete and rigorous overall self consistency of ECE and ECE2 theories.

The Lorentz factor in Eq. (49) is defined by:

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad -(50)$$

in a space with finite torsion and curvature, the mathematical space of ECE2 relativity. Note carefully that the similar looking Lorentz factor of special relativity is defined in a space with zero torsion and zero curvature. In plane polar coordinates the velocity of the Lorentz factor is:

$$v^2 = \dot{r}^2 + r^2 \dot{\phi}^2 \quad -(51)$$

From Eq. (44):

$$\dot{r} = -\frac{L}{m\gamma} \frac{d}{d\phi}\left(\frac{1}{r}\right) \quad -(52)$$

and from Eq. (43):

$$\dot{\phi} = \frac{L}{\gamma m r^2} \quad -(53)$$

so:

$$v^2 = \frac{L^2}{\gamma^2 m^2}\left(\frac{1}{r^2} + \left(\frac{d}{d\phi}\left(\frac{1}{r}\right)\right)^2\right) \quad -(54)$$

It follows that:

$$\frac{1}{\gamma^2} = 1 - \frac{L^2}{m^2 c^2}\left(\frac{1}{r^2} + \left(\frac{d}{d\phi}\left(\frac{1}{r}\right)\right)^2\right) \quad -(55)$$

From Eqs. (49) and (55)

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha}\left(1 - \frac{L^2}{m^2 c^2}\left(\frac{1}{r^2} + \left(\frac{d}{d\phi}\left(\frac{1}{r}\right)\right)^2\right)\right)^{-\frac{1}{2}} \quad -(56)$$

which is the ECE2 covariant equation of orbits with inverse square law of attraction, Q. E. D.

Eq. (56) is a precise analytical equation for the orbit and is derived for the first time in this paper.

It is a non linear second order differential equation of the type:

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{1}{\alpha}\left(1-x\right)^{-1/2} \quad -(57)$$

where:

$$x=\frac{L^2}{m^2c^2}\left(\frac{1}{r^2}+\left(\frac{d}{d\phi}\left(\frac{1}{r}\right)\right)^2\right) \quad -(58)$$

computer algebra can be used to find whether or not it has an analytical solution. If not it can be integrated numerically. From UFT401 it must give orbital precession, a major advance in understanding. If:

$$x \ll 1 \quad -(59)$$

it reduces to:

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{1}{\alpha}\left(1+\frac{x}{2}\right) \quad -(60)$$

and computer algebra can again be used to test whether Eq. (60) has an analytical solution.

The static ellipse is given by:

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{1}{\alpha} \quad -(61)$$

so the precession is given by the additional term on the right hand side of Eq. (60).

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