

```
(%i1) kill(all);
(%o0) done
```

1 Solve eqs.(16,17)

```
(%i1) E16: v[r,P] = (1+Omega[101])*v[r,N]+Omega[102]*v[theta,N];
      E17: v[theta,P] = (1+Omega[202])*v[theta,N]+Omega[201]*v[r,N];
(%o1)  $v_{r,P} = \Omega_{102} v_{\theta,N} + (\Omega_{101} + 1) v_{r,N}$ 
(%o2)  $v_{\theta,P} = (\Omega_{202} + 1) v_{\theta,N} + \Omega_{201} v_{r,N}$ 
```

```
(%i3) solve([E16, E17], [v[r,N], v[theta,N]]);
(%o3) [ [  $v_{r,N} = -\frac{(\Omega_{202} + 1) v_{r,P} - \Omega_{102} v_{\theta,P}}{-\Omega_{202} + \Omega_{101} (-\Omega_{202} - 1) + \Omega_{102} \Omega_{201} - 1}$ ,  $v_{\theta,N} =$ 
 $\frac{-\Omega_{101} v_{\theta,P} - v_{\theta,P} + \Omega_{201} v_{r,P}}{-\Omega_{202} + \Omega_{101} (-\Omega_{202} - 1) + \Omega_{102} \Omega_{201} - 1}$  ] ]
```

```
(%i4) S: ratsimp(%);
(%o4) [ [  $v_{r,N} = -\frac{\Omega_{102} v_{\theta,P} + (-\Omega_{202} - 1) v_{r,P}}{(\Omega_{101} + 1) \Omega_{202} - \Omega_{102} \Omega_{201} + \Omega_{101} + 1}$ ,  $v_{\theta,N} =$ 
 $\frac{(\Omega_{101} + 1) v_{\theta,P} - \Omega_{201} v_{r,P}}{(\Omega_{101} + 1) \Omega_{202} - \Omega_{102} \Omega_{201} + \Omega_{101} + 1}$  ] ]
```

2 Approximations

```
(%i5) Omega[102]: Omega[201]: 0;
(%o5) 0
```

```
(%i6) S1: factor(ev(S));
(%o6) [ [  $v_{r,N} = \frac{v_{r,P}}{\Omega_{101} + 1}$ ,  $v_{\theta,N} = \frac{v_{\theta,P}}{\Omega_{202} + 1}$  ] ]
```

2.1 Solving for the spin connections

```
(%i7) S2: expand(solve(first(S), [Omega[101], Omega[202]]));
(%o7) [ [  $\Omega_{101} = \frac{v_{r,P}}{v_{r,N}} - 1$ ,  $\Omega_{202} = \frac{v_{\theta,P}}{v_{\theta,N}} - 1$  ] ]
```

2.2 Inserting ratio of velocities

```
(%i8) v[r,P]: omega*'diff(r[P],theta);
      v[theta,P]: omega*r[P];
(%o8)  $\omega \left( \frac{d}{d\theta} r_P \right)$ 
(%o9)  $\omega r_P$ 
```

```
(%i10) v[r,N]: omega*'diff(r[N],theta);
      v[theta,N]: omega*r[N];
(%o10)  $\omega \left( \frac{d}{d\theta} r_N \right)$ 
(%o11)  $\omega r_N$ 
```

```
(%i12) S3: ev(S2);
(%o12) [ [  $\Omega_{101} = \frac{\frac{d}{d\theta} r_P}{\frac{d}{d\theta} r_N} - 1$ ,  $\Omega_{202} = \frac{r_P}{r_N} - 1$  ] ]
```

Inserting precessing orbit

```
(%i13) r[N]: alpha/(1+epsilon*cos(theta));
(%o13)  $\frac{\alpha}{\epsilon \cos(\theta) + 1}$ 
```

```
(%i14) r[P]: alpha/(1+epsilon*cos(x*theta));
(%o14)  $\frac{\alpha}{\epsilon \cos(\theta x) + 1}$ 
```

```
(%i15) S4: ev(S3,diff);
(%o15) [ [  $\Omega_{101} = \frac{(\epsilon \cos(\theta) + 1)^2 x \sin(\theta x)}{\sin(\theta) (\epsilon \cos(\theta x) + 1)^2} - 1$ ,  $\Omega_{202} = \frac{\epsilon \cos(\theta) + 1}{\epsilon \cos(\theta x) + 1} - 1$  ] ]
```

Further approximations

```
(%i16) ev(S4, [cos(x*theta)=cos(theta),sin(x*theta)=sin(theta)]);
(%o16) [ [  $\Omega_{101} = x - 1$ ,  $\Omega_{202} = 0$  ] ]
```

```
(%i17) ev(S4, [cos(x*theta)=cos(theta),sin(x*theta)=x*(theta),sin(theta)=theta]);
(%o17) [ [  $\Omega_{101} = x^2 - 1$ ,  $\Omega_{202} = 0$  ] ]
```